Hysteresis in a high-temperature superconductor

1992 J. Phys.: Condens. Matter 4 3813

(http://iopscience.iop.org/0953-8984/4/14/012)

J. Phys.: Condens. Matter 4 (1992) 3813–3818. Printed in the UK

Hysteresis in a high-temperature superconductor

I Edmond† and L D Firth
Physics Department, Paisley College, High Street, Paisley PA1 2BE, UK

Received 1 October 1991, in final form 16 December 1991

Abstract. Magnetic hysteresis measurements have been made at 77 K on a bulk cylindrical specimen of $YBa_2Cu_3O_{7-\delta}$ in fields of up to about 5 mT. Remanent magnetization, coercive force, and energy loss are examined as a function of AC axial field amplitude, and the energy loss was also measured in different superimposed steady fields. Results are discussed in terms of the Anderson–Kim critical-state model. The analytic form of the magnetization is presented for the Anderson–Kim model with a cylindrical specimen.

1. Introduction

Magnetism in superconductors is caused by the distribution of induced supercurrents. For type-II materials, critical state models have had some success in ascribing the magnetism to assumed supercurrent distributions. In such models, the local supercurrent is directed along the vector product of the local flux and the present or last previous flux velocity, but is independent of the magnitude of the flux velocity. A reversal of local flux movement thus causes a reversal of local supercurrent, but surface supercurrents can shield the interior of a specimen from the effects of changing the applied field. Thus a superconductor in an oscillating field can simultaneously contain sheets of oppositely directed current at different depths.

If a specimen is subjected to an external field increasing from zero to a maximum, and then returning to zero, the direction of current induced by the initial increase may be preserved below a surface layer of reversed currents. The magnetic effects of the oppositely directed current sheets do not completely cancel, leaving a remanent magnetization. To reduce the magnetization to zero, a certain oppositely directed field, the coercive force, must be applied. A non-zero remanence and coercive force are characteristic of a hysteresis loop, and the terminology is of course borrowed from the language of ferromagnetism. Although the superconducting response is essentially diamagnetic, the superconducting hysteresis loop is described in the same sense as the ferromagnetic loop, i.e. anticlockwise, since energy is dissipated in both cases.

The M–H hysteresis loop of a ferromagnet is a property of the material, but for a superconducting specimen it is both shape and size dependent. The shape dependence arises because the magnetic moment of a macroscopic current sheet is proportional to the area it encloses. In a cylindrical specimen, this area shrinks more rapidly with depth below the surface than in a slab specimen. It is convenient to account for the size dependence by expressing the average magnetization in terms

† Now at Marconi Defence Systems Ltd, The Grove, Warren Lane, Stanmore HA7 4LY, UK.

of the *penetration field* H^*, which is the surface value of the field which decreases monotonically to zero at the sample centre.

A hysteretic magnetic response is necessarily non-linear, and so a superconductor subjected to a sinusoidal AC field will show harmonics in its magnetization. In the Bean critical-state model, a constant supercurrent density was used (Bean 1964) to account for the odd harmonics observed in conventional type-II superconductors. However, when a high-temperature superconductor (HTSC) is subjected to oscillating fields with a DC bias, both odd and even harmonics are observed (see, e.g., Dorman and Shaulov 1988). The Bean model predicts the same shape of hysteresis loop when a DC field is added, and so fails to explain the loss of symmetry between the two halves of the cycle that the appearance of even harmonics implies. Ji *et al* (1989) showed that a field-dependent supercurrent would explain the appearance of even harmonics. They analysed a particular case—the Anderson–Kim model in a flat-slab geometry. In this model (Anderson and Kim 1964), the supercurrent density is taken to be inversely proportional to the local flux density. This is equivalent to assuming that the supercurrent is just below the value that would overcome a constant pinning force, since the force per unit volume between the current and the flux is the vector product of their respective densities.

In ceramic superconductors the flux penetration described by the Anderson–Kim model occurs between grains and along grain boundaries, the currents in neighbouring grains being weakly linked. The grains themselves resist flux penetration much more strongly. In high fields they contribute significantly to the energy loss; Müller (1991) used the Bean model (cylindrical) to describe their effect. However, in this paper we are concerned with fields some two orders of magnitude smaller; it is then a fair approximation to assume that the grains are in a perfect Meissner state. Their diamagnetism will affect the shape of the experimental hysteresis loops, but not the remanence or the energy loss.

Although the generation of harmonics is undoubtedly of interest, it has diverted attention somewhat from the physical properties of the hysteresis loop itself. In this paper we attempt to redress the balance by comparing the hysteresis parameters in the Anderson–Kim model with measured values.

2. Theory

The Anderson–Kim model has been well described by Ji *et al* (1989). They give a complete analytical solution for the flat-slab geometry (with the field applied in the plane of the slab). However, the solution for the more common cylindrical geometry has not been published; we give it, using SI notation, in the Appendix. The penetration field is given by $H^* = (\alpha d/\mu_0)^{1/2}$ where α is the pinning force per unit volume and d the diameter.

The differences from the flat-slab case are not striking. The remanence reaches a maximum of $8H^*/15$ (compared with $2H^*/3$ for the flat slab) because of the smaller relative volume of deep-lying material in the cylindrical case. The coercive force and the shape (but not the area) of the hysteresis loop depend on the grains remaining in the Meissner state. We have assumed that these grains contribute a term $\mu_0 \chi H$ to the total magnetization, $\chi = -f$ (where f is the grain volume fraction) being the effective grain diamagnetic susceptibility, so $M = M_{\mathrm{AK}} + \mu_0 \chi H$.

This approach, which follows that of Ji *et al* , implies that the Anderson–Kim flux density parameters that we measure are macroscopic averages rather than true local values, but it is impossible to do better without making *ad hoc* assumptions about the grain geometry. Figure 1 shows loops in zero DC field for $\chi = -0.33$ (which gives close agreement with the experiment described below). The following features are independent of the value of χ chosen.

(i) For $H_0 \lesssim H^*/5$, where H_0 is the amplitude of the alternating field, the dissipation is extremely small, because large current densities are possible just below the surface. In this region, the whole specimen is approximately in the Meissner state.

(ii) For $H^*/5 < H_0 < H^*$, dissipation rises rapidly with increasing field amplitude as the high current densities are driven deeper into the specimen, allowing greatly increased flux penetration.

(iii) For $H_0 > H^*$, the alternating field penetrates to the centre of the specimen. The field profile at maximum applied field is completely destroyed at a later time, so loops of different field amplitudes have common sections. The remanence and the coercive force saturate, and the dissipation rises less rapidly with field amplitude.

Further features of the model are presented in the following section along with experimental data.

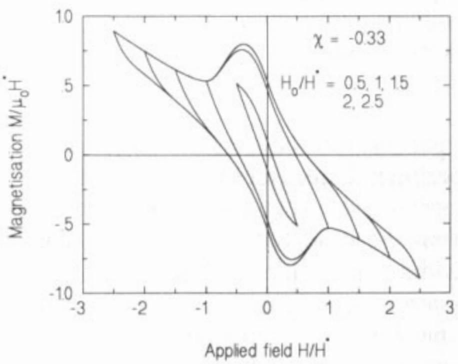

Figure 1. Hysteresis loops predicted by the Anderson–Kim model with cylindrical symmetry, with an additional diamagnetic contribution. The contribution to magnetization from the interior of the specimen is assumed to be zero if $H_0 < H^*$.

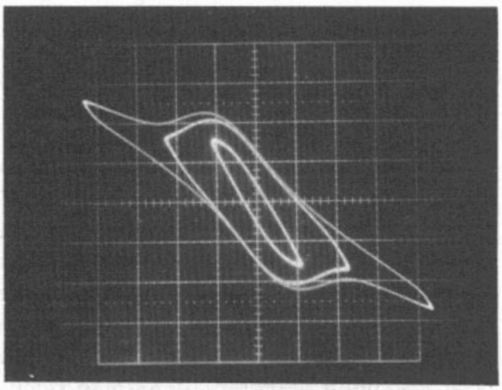

Figure 2. Oscilloscope traces of hysteresis loops of the HTSC specimen. The amplitudes of the alternating flux densities applied are 1.14, 2.37 and 4.54 mT.

3. Experimental method and results

The specimen used was a cylindrical pellet of $YBa_2Cu_3O_{7-\delta}$ ceramic, 7.5 mm in diameter and 8.1 mm in length. It was positioned within similar concentric solenoids carrying DC and 200 Hz AC currents, and within one of an inner pair of pick-up coils, each of 236 turns. The specimen and coil assembly was immersed in liquid nitrogen at 77 K. The pick-up coils were counterwound, and carefully positioned to give zero output in the absence of a specimen. Their output was integrated electronically to give a voltage proportional to the magnetization. Hysteresis loops for various values of AC and DC currents were displayed on an oscilloscope and also captured by a microcomputer. The dissipation was calculated from the stored data by a computer program. Some measurements of remanence and coercive force were made directly from the CRO screen; these were consistent with the data recorded by the computer.

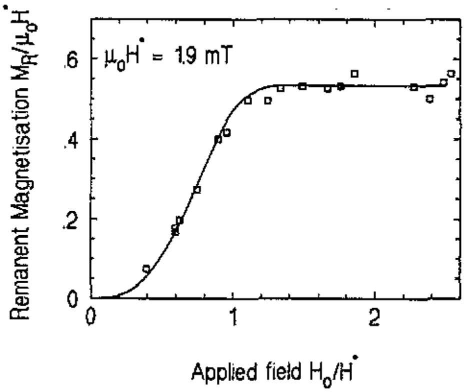

Figure 3. The remanent magnetization of the HTSC specimen as a function of alternating field amplitude (zero DC field). The solid line is the prediction of the Anderson–Kim model.

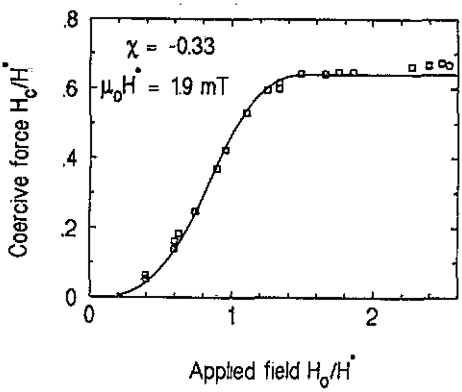

Figure 4. The coercive force for the HTSC specimen as a function of alternating field amplitude (zero DC field). The solid line is the prediction of the Anderson–Kim model with an additional diamagnetic contribution.

Figure 2 is a triple-exposure oscilloscope photograph showing the features described in the theory section. In figure 3 experimental measurements of remanent magnetization have been fitted to the theoretical curve using the single parameter H^*. The value $\mu_0 H^* = 1.9$ mT was found from the observed remanence of about 1.0 mT, which is $8\mu_0 H^*/15$ according to the cylindrical Anderson–Kim model. To fit the coercive force data to the model it is necessary to assume a value of χ, as shown in figure 4. The quality of the fit in figures 3 and 4 may be surprising in view of the poor specimen geometry (length to diameter ratio = 1.08). However, the coercive force is measured at zero average magnetization, eliminating the self-field, and the magnetization is in most cases at a maximum well below the surface (especially when H is close to H^*), so much of the time the effective specimen geometry is quite good.

Figure 5 shows that the Anderson-Kim model accounts well for the variation of dissipation with alternating field amplitude, but over estimates the magnitude (with the assumed value of H^*) by some 10%. The slight discrepancy between the loop shapes in figures 1 and 2 is probably relevant here. Figure 6 shows that further discrepancies are evident when a steady field is superimposed; while the qualitative agreement is good, there is clearly quantitative disagreement beyond experimental uncertainty. Geometric effects are a possible cause, but so also is dissipation within the grains which become significant for $H/H^* > 2$ as figure 4 suggests.

In conclusion, we believe that the hysteresis loops of the HTSC specimen in the range of fields studied are described well by the Anderson–Kim model if an additional diamagnetic component is included in the magnetization to allow for the interiors of

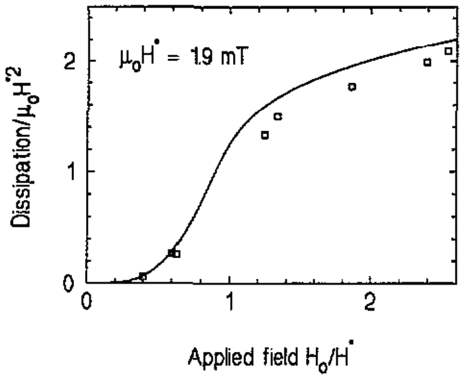

Figure 5. Dissipation per cycle of the HTSC specimen as a function of alternating field amplitude (zero DC field). The solid line is the prediction of the Anderson–Kim model.

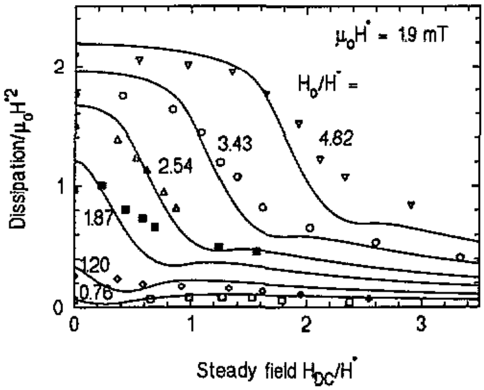

Figure 6. Dissipation per cycle of the HTSC specimen as a function of the superimposed steady field, for the values of alternating field amplitudes shown. The solid lines are the predictions of the Anderson–Kim model for the same values of alternating fields.

the grains. The fits for the coercive force and remanent magnetization are particularly good. The detailed shapes of the loops, and consequently the dissipation in AC and AC plus DC fields show less satisfactory agreement with experiment, but still at least as good as that found by Müller using a more elaborate version of the Anderson–Kim model. The penetration field for the specimen of 1.9 mT corresponds to a flux pinning force per unit volume of 383 N m^{-3}.

Appendix

Following Ji *et al* (1989) we have calculated the volume average $\boldsymbol{B}$ of the flux density in a long cylindrical superconductor subject to an applied axial field $\boldsymbol{H}$ cycled between upper and lower limits H_u and H_l. We define reduced fields in terms of the penetration field H^*:

$$h = H/H^* \qquad h_u = H_u/H^* \qquad h_l = H_l/H^* \qquad b = B/\mu_0 H^*$$

and the functions

$$f(u,v) \equiv \tfrac{1}{2}(u^2 \operatorname{sgn}(u) - v^2 \operatorname{sgn}(v)) \qquad g(u,v) \equiv |f(u,-v)|^{3/2}(1 - |f(u,v)|).$$

The value of $\Delta^2 \equiv f(h_u, h_l)$ determines whether flux moves into the specimen centre. Any volume that is completely shielded from the changing field can provide a contribution $B_0 = \mu_0 H^* b_0$ to the average flux density which is determined by the *initial conditions.*

We also define

$$b_1 \equiv (1-\Delta^2)|f(h_u^2, -h_l|^{1/2} + \tfrac{8}{15}|f(h_u, -h_l|^{5/2} \operatorname{sgn}(h_u^2 - h_l^2).$$

We give below the expressions for the reduced mean flux densities under the various conditions that may apply. The mean magnetization in the Anderson–Kim model is obtained in each case from $M_{AK} = B/\mu_0 - H = H^*(b - h)$.

A1. For H decreasing

(i) If $\Delta^2 < 1$ then

$$b = \tfrac{8}{3}g(h_u, h) - \tfrac{4}{3}g(h_u, h_l) - \tfrac{4}{3}|h|^3 - \tfrac{8}{15}h^5 + b_1 + b_0.$$

(ii) If $\Delta^2 > 1$, if

$$h > |h_u^2 \operatorname{sgn}(h_u) - 2|^{1/2} \operatorname{sgn}(h_u^2 \operatorname{sgn}(h_u) - 2)$$

then

$$b = \tfrac{8}{3}g(h_u, h) - \tfrac{4}{3}|h|^3 - \tfrac{8}{15}h^5 + \tfrac{8}{15}|h_u^2 - \operatorname{sgn}(h_u)|^{5/2} \operatorname{sgn}(h_u - 1)$$

and otherwise

$$b = -\tfrac{4}{3}|h|^3 - \tfrac{8}{15}h^5 + \tfrac{8}{15}|h^2 + \operatorname{sgn}(h)|^{5/2} \operatorname{sgn}(h + 1).$$

A2. For H increasing

(i) If $\Delta^2 < 1$ then

$$b = -\tfrac{8}{3}g(h_l, h) + \tfrac{4}{3}g(h_u, h_l) + \tfrac{4}{3}|h|^3 - \tfrac{8}{15}h^5 + b_1 + b_0.$$

(ii) If $\Delta^2 > 1$, if

$$h < |h_l^2 \operatorname{sgn}(h_l) + 2|^{1/2} \operatorname{sgn}(h_l^2 \operatorname{sgn}(h_l) + 2)$$

then

$$b = -\tfrac{8}{3}g(h_l, h) + \tfrac{4}{3}|h|^3 - \tfrac{8}{15}h^5 + \tfrac{8}{15}|h_l^2 + \operatorname{sgn}(h_l)|^{5/2} \operatorname{sgn}(h_l + 1)$$

and otherwise

$$b = \tfrac{4}{3}|h|^3 - \tfrac{8}{15}h^5 + \tfrac{8}{15}|h^2 \operatorname{sgn}(h)|^{5/2} \operatorname{sgn}(h - 1).$$

References

Anderson P W and Kim Y B 1964 *Rev. Mod. Phys.* **36** 39
Bean C P 1964 *Rev. Mod. Phys.* **36** 31
Dorman D and Shaulov A 1988 *Appl. Phys. Lett.* **53** 2680
Ji L, Sohn R H, Spalding G C, Lobb C J and Tinkham M 1989 *Phys. Rev.* B **40** 1093
Müller K-H 1991 *IEEE Trans. Magn.* MAG-27 2174